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# Transonic Similarity Theory Applied to a Supercritical Airfoil in Heavy Gas

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**The use of a high-molecular-weight test gas to increase the Reynolds number range of transonic wind tunnels is explored experimentally. Modifications to a transonic wind tunnel for heavy-gas operation are described, and the real-gas properties of the example heavy gas (sulfur hexafluoride) are discussed. Sulfur hexafluoride is shown to increase the test Reynolds number by a factor of more than 2 over air at the same stagnation conditions and test section Mach number. Experimental and computational pressure distributions on an advanced supercritical airfoil at Mach numbers of 0.7 and 0.72 in both sulfur hexafluoride and nitrogen are presented. Transonic similarity theory is shown to be successful in transforming the heavy-gas results to equivalent nitrogen (air) results, provided the correct definition of gamma is used and viscous effects are not dominant. When strong shocks are present on the airfoil upper surface, transonic similarity theory is shown to be less successful in the shock–boundary-layer interaction region, in agreement with computational predictions.**

## Nomenclature

$A$ = similarity parameter; Eq. (11)
$a$ = speed of sound
$a_i, b_i, c_i$ = constants in equation of state; Eq. (1)
$C_l$ = lift coefficient
$C_p$ = pressure coefficient; $(p - p_{\mathrm{ref}})/q_\infty$
$c$ = airfoil chord
$c_p$ = specific heat at constant pressure
$c_v$ = specific heat at constant volume
$d$ = constant in equation of state; Eq. (1)
HP = drive horsepower
$h$ = enthalpy
$M$ = Mach number
MWt = molecular weight
$p$ = pressure
$p_{\mathrm{ref}}$ = static pressure at entrance to test section
$q$ = dynamic pressure
$R$ = specific gas constant
$Re$ = Reynolds number
$S$ = entropy
$T$ = temperature
$T_c$ = ref. temperature, 318.8 K; Eq. (1)
$t$ = airfoil thickness
$x$ = streamwise distance on model
$\alpha$ = angle of attack
$\Gamma$ = fundamental derivative; Eq. (8)
$\gamma$ = ratio of specific heats
$\mu$ = viscosity
$v$ = specific volume
$\tau$ = airfoil thickness parameter, $t/c$
$\chi$ = similarity parameter

*Subscripts*

air = air
hg = heavy gas
$\infty$ = freestream conditions

## Introduction

THE shortfall in Reynolds number in current U.S. transonic ground test facilities is well known (Mack et al.[1]), and with the advent of the proposed, so-called megaliners (600–800 passenger transonic transport aircraft), this shortfall is likely to grow significantly in the coming years. Test data needed by the designers of these large-scale aircraft will require higher Reynolds number test facilities, but recent engineering estimates indicate that large scale, high Reynolds number, transonic wind tunnels will be expensive to build and to operate. All of this has occurred in an era of increased commercial competition to reduce the design cycle time and to lower the cost of developing new transport aircraft, and it is compounded by declining research budgets. The only facilities currently capable of producing near-flight-level Reynolds numbers at transonic speeds are cryogenic tunnels. Two such tunnels in the U.S. and two in Europe are available for commercial use. These cryogenic facilities rely on a combination of increased pressure and reduced temperature to increase the density and reduce the viscosity of the test gas (nitrogen). This technique can produce freestream unit Reynolds numbers in excess of $100 \times 10^6$/ft and, augmented by the additional benefit that the lower speed of sound at cryogenic temperatures results in reduced drive horsepower and model loads, offers the only currently practicable solution for high Reynolds number testing. However, experience has shown that wind-tunnel testing at cryogenic temperatures ($\approx$100 K) is expensive and far from routine, requiring special care with regard to instrumentation and model construction. Consequently, because of the high cost and difficulty associated with cryogenic testing, and because it is unlikely that additional, large-scale cryogenic facilities will be constructed to relieve the crowded test schedules in the existing tunnels, NASA has initiated research into alternative techniques for increasing test Reynolds numbers.

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One technique that may hold promise is that of increasing the Reynolds number through the use of a high-molecular-weight gas as the test medium. The advantages of such a technique are clear. Testing can be done at ambient temperatures, and existing closed-circuit wind tunnels can be retrofitted to use "heavy gas." This latter advantage becomes even more significant in view of the high cost of constructing new facilities.

The use of heavy gas to increase Reynolds number is an old idea dating back to work by Huber,[2] Schwartzberg,[3] vonDoenhoff et al.,[4] and Chapman.[5] Chapman's work was an exhaustive examination of candidate gases that could be combined to produce gas mixtures with a ratio of specific heats ($\gamma$) near 1.4. Later work by Pozniak,[6] Treon, et al.,[7] Yates and Sandford,[8] and others focused on correlating results in gases where $\gamma$ deviated from 1.4 to equivalent results in air. Results from these earlier studies were inconclusive, and the technique, at least for aerodynamic purposes, has lain essentially dormant for the past 20 years. However, these early studies did serve to highlight the central problem with heavy gas; how does one relate results in a test gas in which $\gamma$ is substantially different from air, and that may exhibit large caloric and thermal imperfections, to equivalent results in air? Anderson,[9] in 1991, conducted a computational study of transonic flow over airfoils in sulfur hexafluoride ($SF_6$) and found that transonic similarity scaling could be used to relate results in $SF_6$ to equivalent air results as long as the proper definition of $\gamma$ was used. Anderson also found that as viscous effects became more pronounced, and the flow became more compressible, the scaling was somewhat less effective. Later, Bonhaus et al.[10] computed the flow over two multi-element high-lift configurations in $SF_6$ and concluded that transonic similarity scaling was adequate for such flows.

Following Anderson's computational demonstration of the concept, NASA initiated a pilot experimental program to study the feasibility of transonic testing with heavy gas. The objective was to experimentally verify the transonic similarity scaling by testing a representative supercritical airfoil, both in a conventional test medium (air/nitrogen) and in heavy gas. To make meaningful comparative studies, it was necessary to conduct the tests at the same Reynolds number in both gases and, preferably, under conditions of low wall interference on the model surface pressures. The NASA Langley 0.3-Meter Transonic Cryogenic Tunnel (0.3-M TCT) was chosen to implement and experimentally demonstrate this proof-of-concept study. The advantages of using the 0.3-M TCT included the possibility of making comparative studies at high Reynolds numbers under nearly identical interference conditions by using the streamlining capability of the adaptive wall test section. This paper discusses the results of that experimental study, which used a representative supercritical airfoil in both cryogenic nitrogen gas and in $SF_6$ at transonic speeds. Transonic similarity theory is used to determine the test Mach numbers, and measured airfoil pressure distributions are compared with computational results for both gases. For the purpose of this comparative study, air and nitrogen are assumed to be equivalent gases.

## Methods

### Experimental Facility

The test facility used for the current study (0.3-Meter TCT at the Langley Research Center) is described in some detail in Mineck and Hill.[11] This facility was initially developed to demonstrate the cryogenic testing concept, and this development eventually led to the construction of NASA's National Transonic Facility. The 0.3-M TCT operates at pressures of up to 6 atm, temperatures from 100 to 300 K, and Mach numbers from 0.15 to 1.0. The test section is approximately 13 × 13 in., and the upper and lower walls are adaptive with self-streamlining capability. The modifications required for operation with $SF_6$ included a gas-reclamation unit for charging and reclaiming the test gas from the tunnel, a gas-analysis unit for real-time monitoring of gas composition, a gas-warning system for personnel safety, and a specially designed heat exchanger. These modifications are discussed in Anders,[12] and with the exception of the custom-designed heat exchanger, all of these units are off the shelf, commercially available systems. Figure 1 shows a schematic of the modified tunnel with the additional systems required for heavy gas operation denoted. Figure 2 shows the operating envelope of the tunnel. Although the Reynolds number range for $SF_6$ is approximately half that of cryogenic nitrogen, the upper limit approaches $60 \times 10^6$/ft, substantially greater than that for ambient temperature air operation.

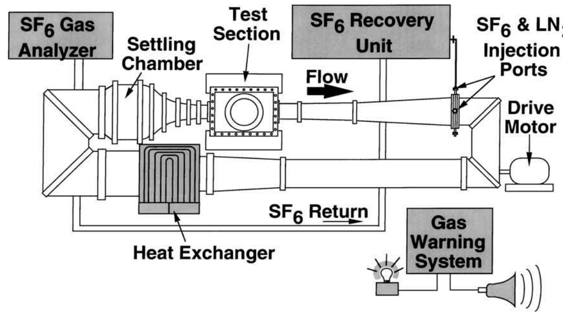



**Fig. 1 Schematic of 0.3-M TCT with modifications for heavy-gas operation (shaded areas indicate heavy-gas components).**

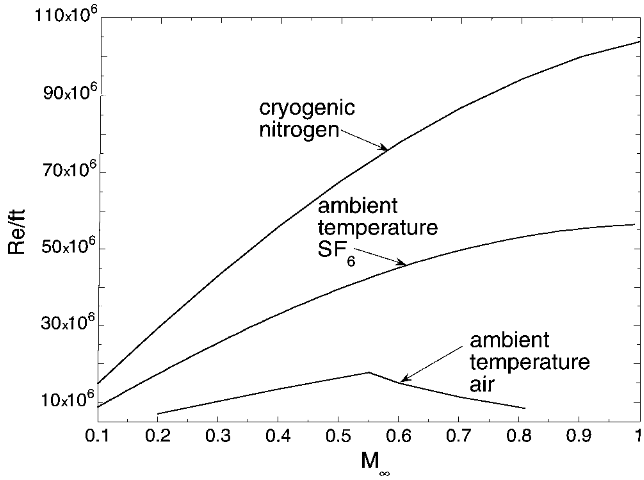



**Fig. 2 Operating envelopes for 0.3-M TCT.**

### Model

The model used for the current study was an advanced supercritical airfoil with 78 pressure taps over the upper and lower surfaces, mounted between sidewall turntables, and spanning the test section width. The model was tested near design lift conditions in the angle-of-attack range from +1.0 to −0.75 deg. The test section upper and lower walls were streamlined to minimize tunnel wall interference effects.[11] The majority of the data were taken in the Mach number range from 0.70 to 0.72 at an airfoil chord Reynolds number of $30 \times 10^6$. A limited amount of data were also obtained at $15 \times 10^6$.

The accuracy of the measured static pressures on the airfoil is estimated as ±0.5%. The accuracy of the test Mach number is ±0.002, and the airfoil angle of attack is ±0.05 deg of the set value. Figure 3 shows an example of the repeatability of the airfoil surface pressure measurements in $SF_6$ from three different runs under identical test conditions.

### Characteristics of Sulfur Hexafluoride

Sulfur hexafluoride is a colorless, odorless, nonflammable, nontoxic gas with a molecular weight of 146. Its principal commercial use is as a dielectric in high-voltage switchgear, and it is readily available from a number of manufacturers. It is essentially an inert gas at ordinary laboratory temperatures, although it may be partially decomposed by electric arcs into lower fluorides of sulfur. It has no ozone-depletion potential, but it is a greenhouse gas. $SF_6$ is both calorically and thermally imperfect, but its properties are well known and documented. Equipment for handling $SF_6$ (condensers, vaporizers, storage tanks, pumps, etc.) is manufactured

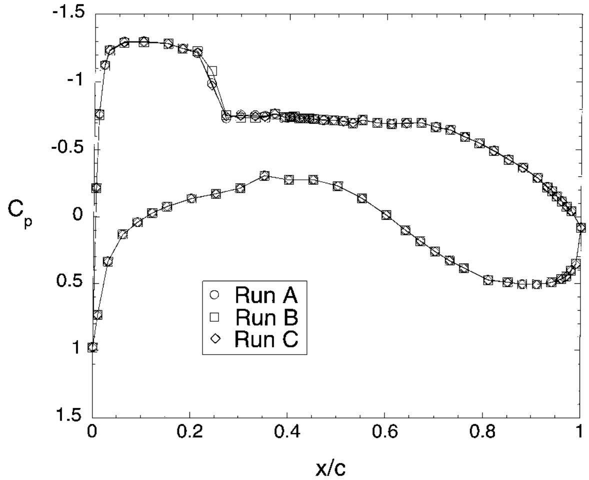



**Fig. 3 Repeatability of $SF_6$ pressure distributions: $\alpha$ = 1.0 deg, $M_\infty$ = 0.72, $Re_c$ = 30 × $10^6$.**

commercially for power companies and is readily available from a number of suppliers. The principal hazard from this gas is that of asphyxiation because it displaces air. The equation of state[9,13] is well represented by

$$p = \frac{RT}{v-d} + \sum_{i=2}^{5} \frac{a_i + b_i T + c_i e^{-kT/T_c}}{(v-d)^i} \tag{1}$$

where the various constants are given as

$$a_2 = -49.9051433\,\mathrm{N\cdot m^4/kg^2}$$

$$b_2 = 5.485495\times 10^{-2}\,\mathrm{N\cdot m^4/kg^2\cdot K}$$

$$c_2 = -2.375924505\times 10^{3}\,\mathrm{N\cdot m^4/kg^2}$$

$$a_3 = 4.124606\times 10^{-2}\,\mathrm{N\cdot m^7/kg^3}$$

$$b_3 = -3.340088\times 10^{-5}\,\mathrm{N\cdot m^7/kg^3\cdot K}$$

$$c_3 = 2.819595\,\mathrm{N\cdot m^7/kg^3}$$

$$a_4 = -1.612953\times 10^{-5}\,\mathrm{N\cdot m^{10}/kg^4}, \qquad b_4 = 0, \qquad c_4 = 0$$

$$a_5 = -4.899779\times 10^{-11}\,\mathrm{N\cdot m^{13}/kg^5}$$

$$b_5 = 1.094195\times 10^{-11}\,\mathrm{N\cdot m^{13}/kg^5\cdot K}$$

$$c_5 = -3.082731\times 10^{-7}\,\mathrm{N\cdot m^{13}/kg^5}, \qquad k = 6.883022$$

$$T_c = 318.8\,\mathrm{K}, \qquad d = 3.27367367\times 10^{-4}\,\mathrm{m^3/kg}$$

Anderson[9] and Anders[12] discuss the real-gas behavior of $SF_6$ and conclude that accounting for the thermal imperfections of the gas is necessary, even though the corrections are not large at typical wind tunnel stagnation pressures. The caloric imperfections, however, are shown to be quite large, and the variable specific heats must be accounted for in calculating the isentropic flow properties. In addition, Anders[12] concludes that small amounts of air and water-vapor contamination of the test gas have little or no effect on the freestream flow properties, but they do affect the mixture viscosity slightly. In the present experiment the test gas composition was continuously monitored during tunnel runs and indicated an $SF_6$ purity of approximately 98%. Jenkins[14] has developed a computer code to calculate the isentropic flow properties of $SF_6$, and this code was used for determining the test flow conditions, adaptive wall operation, and the data reduction of the experimental results presented here.

**Table 1 Gas characteristics**

| Gas | Molecular weight | $\gamma$ | $\mu$ (lb · s/ft$^2$) × $10^8$ | $Re_{hg}/Re_{air}$ |
|---|---|---|---|---|
| Air | 29 | 1.4 | 38.6 | 1 |
| $SF_6$ | 146 | 1.095 | 32.6 | 2.4 |
| R134A | 102 | 1.105 | 28.6 | 2.3 |
| R125 | 120 | 1.109 | 31.3 | 2.3 |

Assuming for the moment that $SF_6$ is a perfect gas, it is easy to show that the Reynolds number increase for $SF_6$ over air at the same Mach number, total pressure, and total temperature can be written as[5]

$$\frac{Re_{hg}}{Re_{air}} \approx \left(\frac{\mu_{air}}{\mu_{hg}}\right)\sqrt{\frac{(\mathrm{MWt})_{hg}}{(\mathrm{MWt})_{air}}} \approx 2.4 \tag{2}$$

Similar expressions can be developed for the dynamic pressure and the required drive horsepower[5] for the same Mach number, Reynolds number, and total temperature:

$$\frac{q_{hg}}{q_{air}} \approx \left(\frac{\mu_{hg}}{\mu_{air}}\right)\sqrt{\frac{(\mathrm{MWt})_{air}}{(\mathrm{MWt})_{hg}}} \approx 0.4 \tag{3}$$

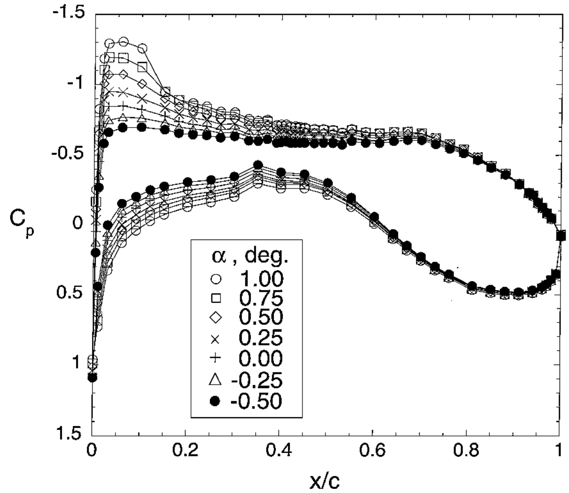
$$\frac{\mathrm{HP}_{hg}}{\mathrm{HP}_{air}} \approx \left(\frac{\mu_{hg}}{\mu_{air}}\right)\left[\frac{(\mathrm{MWt})_{air}}{(\mathrm{MWt})_{hg}}\right] \approx 0.2 \tag{4}$$

Although sulfur hexafluoride was one of the candidate gases originally proposed by Chapman, new refrigerant gases such as R134A (the replacement gas for Freon-12) have become common since that time and may, in fact, be preferable because of environmental (greenhouse) and cost concerns. However, $SF_6$ is representative of the class of high-molecular-weight gases that can, as shown herein, increase the Reynolds number and decrease the dynamic pressure and drive horsepower. These characteristics may be particularly attractive for selected, existing transonic wind tunnels to provide an enhanced Reynolds number test range. Table 1 compares some of the physical properties of $SF_6$ with air and with two of the new refrigerant gases. Technically, the ratio of specific heats is not a constant for the three heavy gases listed in Table 1, but it is treated as such for the purpose of comparison. For all other purposes in this paper, the ratio of specific heats is treated as a variable.

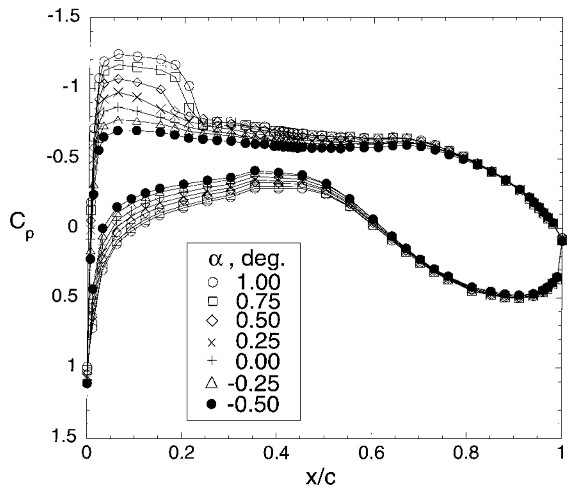
## Results and Discussion

### Experimental Results

Typical pressure distributions over the test airfoil in both $SF_6$ and cryogenic $N_2$ are shown in Figs. 4a and 4b for $M_\infty = 0.70$, $Re_c = 30 \times 10^6$, and $\alpha = -0.5$ to $+1.0$ deg. Depending on the exact location of the stagnation point with respect to the model pressure orifices, the $C_p$ values in the stagnation region can sometimes exceed 1.0, as expected for compressible flow. There are clear differences in the pressure distributions between the two gases, especially when the flow over the airfoil upper surface becomes supercritical. The onset of supercritical flow occurs earlier with increasing angle of attack for nitrogen than for $SF_6$. This is demonstrated in Figs. 5a–5c, which show a direct comparison between the two gases at three angles of attack. The agreement on the lower surface, and on the rear of the upper surface, where the local Mach numbers are subcritical, is fairly good, as would be expected when compressibility effects are smaller. However, on the forward part of the upper surface, as the angle of attack increases and the flow becomes supercritical (the local Mach number for nitrogen approaches 1.3), significant differences appear, especially in the location of the shock. This effect becomes even more pronounced when $M_\infty = 0.72$, as shown in Fig. 6. For identical freestream conditions, the shock on the airfoil upper surface in nitrogen is stronger and is located further downstream compared with $SF_6$ results.

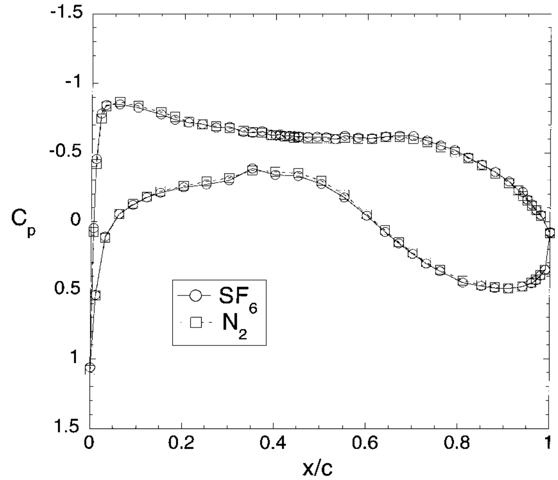



a) $SF_6$

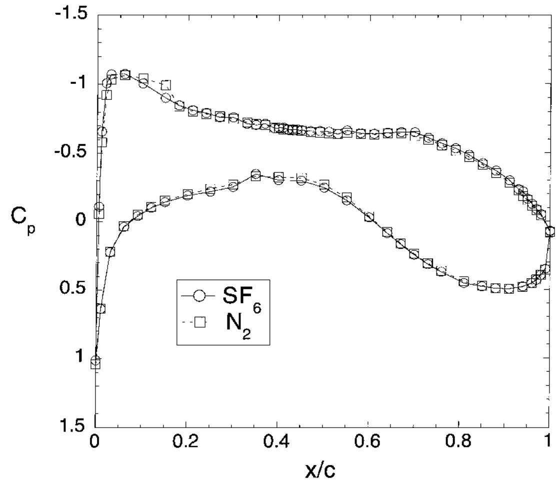



b) $N_2$

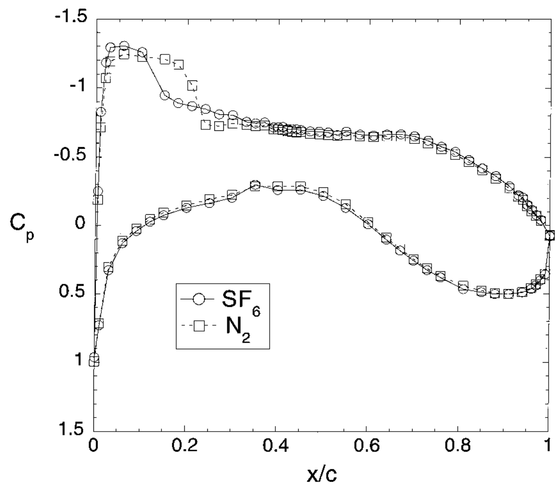
**Fig. 4 Pressure distributions on a supercritical airfoil, with $M_\infty$ = 0.7 and $Re_c$ = 30 × 10$^6$.**




a) $\alpha$ = 0.0 deg




b) $\alpha$ = 0.5 deg




c) $\alpha$ = 1.0 deg

**Fig. 5 $N_2$ and $SF_6$ pressure distributions, with $M_\infty$ = 0.70 and $Re_c$ = 30 × 10$^6$.**

### Transonic Similarity

According to inviscid, small disturbance, transonic similarity theory,[15] the flow around a family of two-dimensional slender profiles is similar if the following parameter is matched:

$$\chi = \left\{ \frac{1 - M_\infty^2}{\left[\tau M_\infty^2 (\gamma + 1)\right]^{\frac{2}{3}}} \right\} \tag{5}$$

Obviously, for flows in air (or any other single gas) a different model ($\tau$) is required for each Mach number in order to maintain similarity. However, for flows in different gases, similarity can be maintained on the same model ($\tau$ = constant) if $\gamma$ is different for the two gases, that is, $\gamma$ and $M_\infty$ become the parameters (with $\tau$ fixed) rather than $\tau$ and $M_\infty$ (with $\gamma$ fixed).

The similarity requirement relating heavy gas to nitrogen (or air) on the same profile then becomes

$$\left\{ \frac{1 - M_\infty^2}{\left[\tau M_\infty^2 (\gamma' + 1)\right]^{\frac{2}{3}}} \right\}_{\text{heavy gas}} = \left\{ \frac{1 - M_\infty^2}{\left[\tau M_\infty^2 (\gamma + 1)\right]^{\frac{2}{3}}} \right\}_{\text{nitrogen}} \tag{6}$$

where $\tau$ is constant and may be eliminated, but $\gamma$ on the left side of Eq. (6) has been replaced by $\gamma'$. Because the ratio of specific heats is not a constant for $SF_6$, it is not immediately obvious what value should be used for $\gamma'$ in Eq. (6). Anderson[9] conducted a detailed study to determine an appropriate definition of $\gamma'$. Out of four different possibilities examined, including the conventional definition for a perfect gas ($c_p/c_v$), the following definition was shown to provide the correct behavior across a wide range of freestream conditions:

$$\gamma' = 2\Gamma_\infty - 1 = 1 + \left(\frac{\delta a^2}{\delta h}\right)_{S,\infty} \tag{7}$$

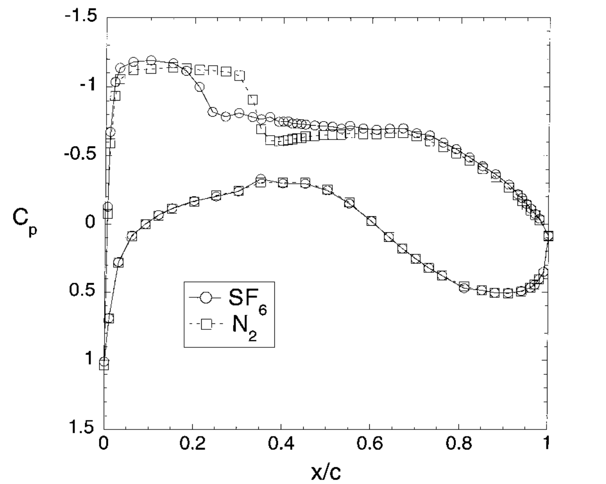



**Fig. 6 $N_2$ and $SF_6$ pressure distributions, with $M_\infty$ = 0.72 and $Re_c$ = 30 × $10^6$.**

The parameter $\Gamma$, known as the fundamental derivative, is defined as[16]

$$\Gamma = \frac{a^4}{2\nu^3}\left(\frac{\delta^2 \nu}{\delta p^2}\right)_s \tag{8}$$

and corresponds to the curvature of an isentrope in the pressure-specific volume plane. For most real fluids $\Gamma$ is positive,[17] and the physical existence of such classical gas-dynamic phenomena as compression shocks ($+\mathrm{d}p/\mathrm{d}x$), rather than rarefaction shocks ($-\mathrm{d}p/\mathrm{d}x$), relies on the assumption that $\Gamma > 1$. For further discussion of the significance of the fundamental derivative $\Gamma$, the reader is referred to Thompson[16] and to Hayes[18]. For a perfect gas with constant specific heats, $\Gamma$ degenerates to $(\gamma + 1)/2$, which results in $\gamma' = \gamma = c_p/c_v$.

At a given freestream Mach number in air, the Mach number in $SF_6$ is determined by first using the freestream pressure and temperature in $SF_6$ to determine $\gamma'$. The new Mach number can then be determined by using Eq. (6). After the fact, the lift and pressure coefficients must be corrected by[9,15]

$$C_{l,\text{nitrogen}} = A C_{l,\text{heavy gas}} \tag{9}$$

$$C_{p,\text{nitrogen}} = A C_{p,\text{heavy gas}} \tag{10}$$

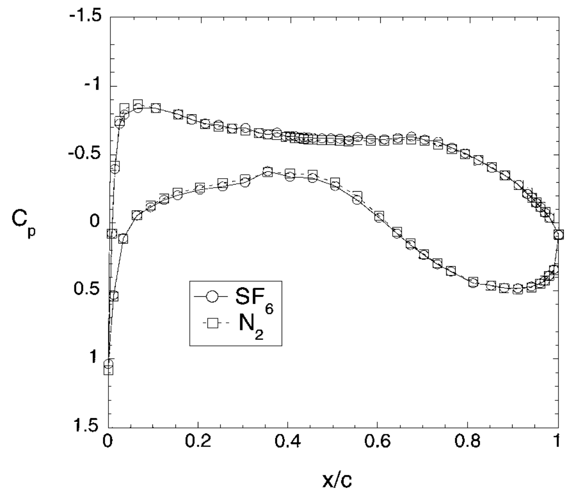
where the parameter $A$ is given by

$$A = \left(\frac{\gamma' + 1}{\gamma + 1}\right)\left(\frac{M^2_{\text{heavy gas}}}{M^2_{\text{nitrogen}}}\right)\left(\frac{1 - M^2_{\text{nitrogen}}}{1 - M^2_{\text{heavy gas}}}\right) \tag{11}$$

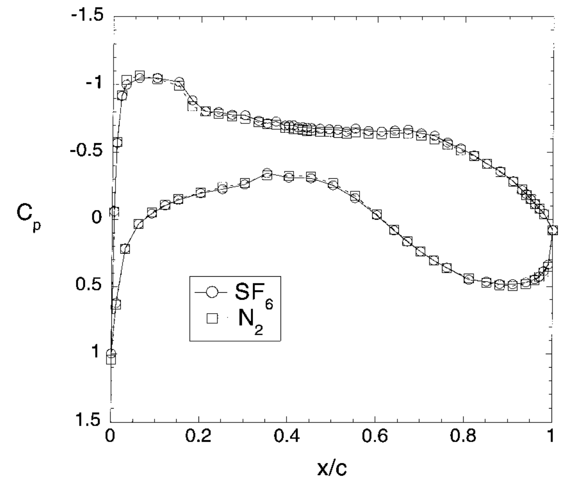
With the use of the above definition of $\gamma'$ in conjunction with the transonic similarity law, inviscid computational results obtained in $SF_6$ have been demonstrated[9] to scale consistently to yield good agreement with computations in air over a wide range of pressures and temperatures. However, the application of these scaling laws to flows with strong shocks was shown to be slightly less successful in the region of the shocks.

### Scaling Applied to Experimental Results

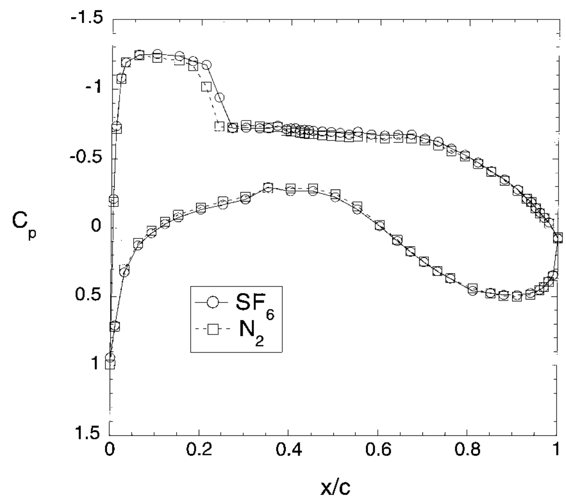
Using the transonic similarity approach described here, it can be shown that for the present experiment the flow past the airfoil at a freestream Mach number of 0.72 in $SF_6$ (where $\gamma' = 1.04$) is equivalent to flow at a freestream Mach number of 0.70 in $N_2$ (where $\gamma = 1.4$). The corresponding value of the scaling parameter $A$ for force and pressure coefficients is computed to be 0.967. Therefore, to examine the validity of the transonic similarity rule for the two gases, test were conducted at $M = 0.70$ for nitrogen and at $M = 0.72$ for $SF_6$, over an angle-of-attack range and at a chord Reynolds number of $30 \times 10^6$ for both gases. The pressure distributions are compared in Figs. 7a–7c for three angles of attack. At $\alpha = 0.0$ and 0.5 deg, when the flow is subcritical (or supercritical with a weak shock) similarity scaling is found to provide good agreement between the two gases. At $\alpha = 1.0$ deg; the pressure distribution over most of the airfoil agrees well, but the shock location for the $SF_6$ case is slightly downstream of the location for $N_2$. Comparisons at $Re_c = 15 \times 10^6$ (not shown) indicate a similar difference in the region of a strong shock. In fact, this difference was so consistent in the experiment results that it probably cannot be attributed to experimental inaccuracy, even though the shock location on supercritical airfoils can




**a) $\alpha$ = 0.0 deg**

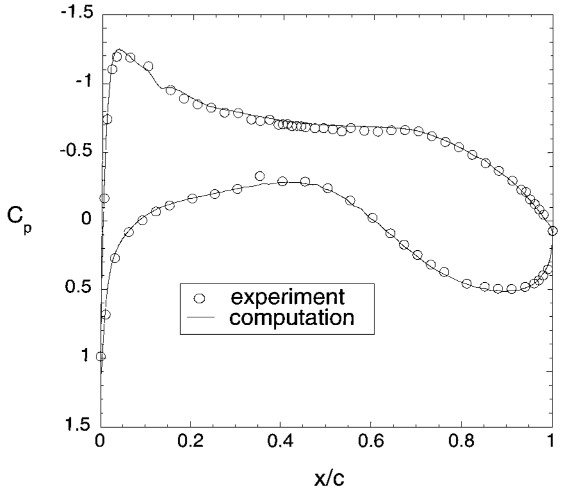



**b) $\alpha$ = 0.5 deg**

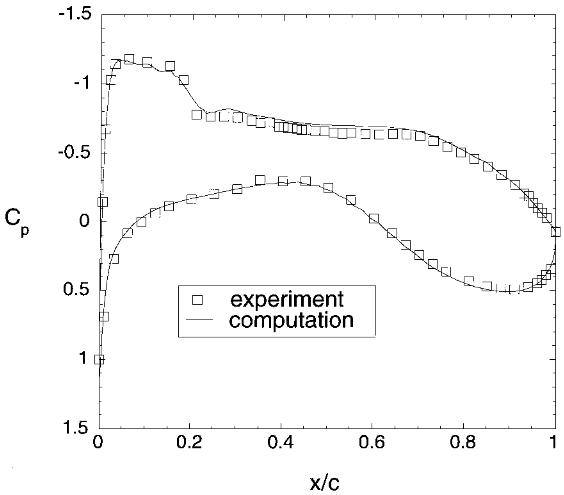



**c) $\alpha$ = 1.0 deg**

**Fig. 7 Transonic similarity scaling applied to experimental results, with $M_{N_2}$ = 0.70, $M_{SF_6}$ = 0.72, and $Re_c$ = 30 × $10^6$.**

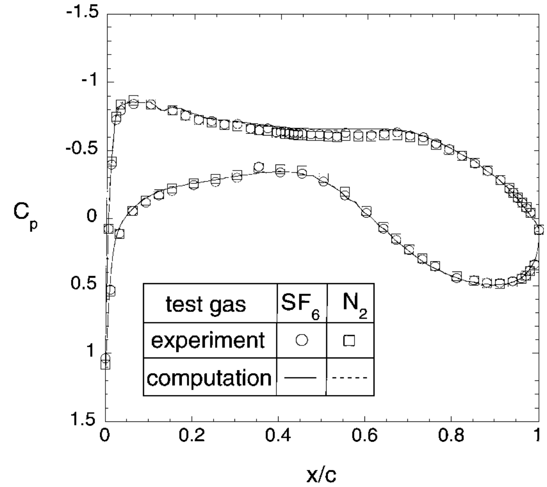



a) $SF_6$

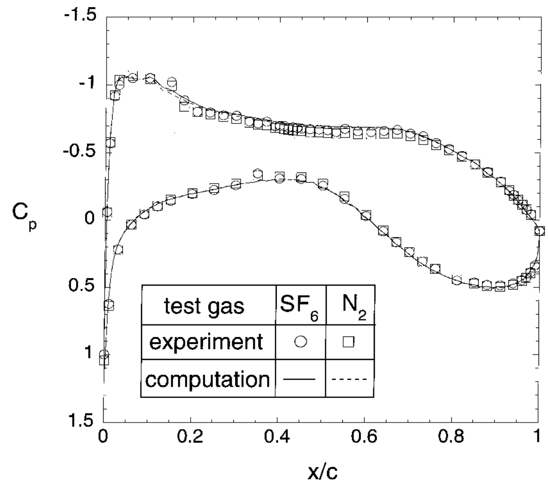



b) $N_2$

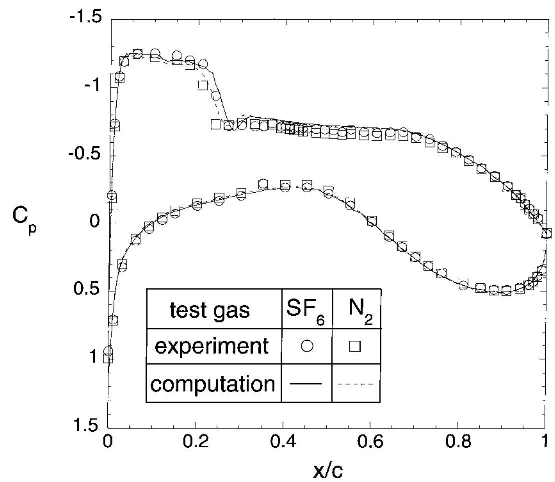
**Fig. 8 Computed and experimental pressure distributions, with $M_\infty$ = 0.70, $Re_c$ = 30 × 10$^6$, and $\alpha$ = 0.75 deg.**




a) $\alpha$ = 0.0 deg




b) $\alpha$ = 0.5 deg




c) $\alpha$ = 1.0 deg

**Fig. 9 Scaled experimental and computational results, with $M_{N_2}$ = 0.70, $M_{SF_6}$ = 0.72, and $Re_c$ = 30 × 10$^6$.**

be highly sensitive to experimental conditions. It is more likely that there are differences in the viscous shock–boundary-layer interaction process between the two gases, and some discussion of this will be included in the subsequent section on viscous similarity. The present results, however, do demonstrate that transonic similarity theory provides a good method of scaling heavy-gas pressure distributions, and it may be adequate for many applications. Clearly, more extensive testing under different test conditions with different airfoil shapes is required before a final determination can be made.

### Computational Method and Results

The computations were done by using an unstructured grid Navier–Stokes solver[19] with suitable modifications to properly account for the nonideal gas behavior of $SF_6$. Figures 8a and 8b show examples of the computational results at $Re_c = 30 \times 10^6$ and $\alpha = 0.75$ deg compared with the measured pressures for both $SF_6$ and $N_2$. The agreement between computation and experiment is considered to be quite good except for a consistent overprediction (more negative $C_p$) of the upper surface pressures downstream of the shock. The computed results in Fig. 8b are actually for air rather than nitrogen. This was done as a matter of convenience, because for all practical purposes the two gases are aerodynamically identical. The agreement shown in Fig. 8a gives some assurance that the real-gas behavior of $SF_6$ is correctly captured in the computations.

Figures 9a–9c show $SF_6$ and $N_2$ experimental results at the similarity Mach number, along with computational results for $SF_6$ and air, again at the similarity Mach number. The agreement between computation and experiment with similarity scaling is considered satisfactory, except for the small, consistent discrepancy in the shock region, especially at the highest angle of attack. In Fig. 10 the vertical scale for the $\alpha = 1.0$ deg case has been expanded to illustrate that the computations show exactly the same trend in the region of the shock as the experimental data. That is, the $SF_6$ calculations at the similarity Mach number predict a shock location slightly downstream of the shock location for air (nitrogen in the case of the experimental data). Also, note that downstream of the shock the pressure

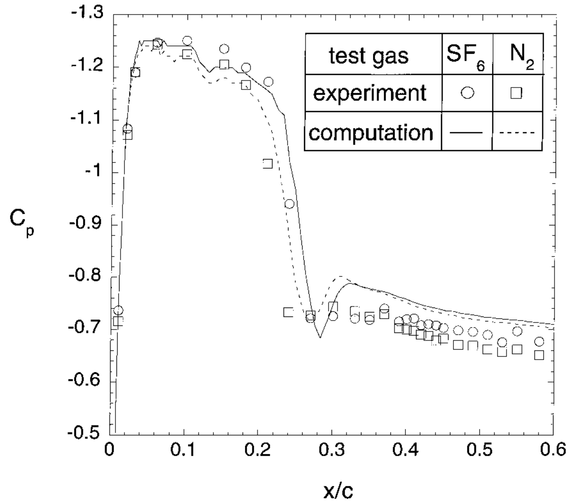



**Fig. 10 Scaled experimental and computational results, with $M_{N_2}$ = 0.70, $M_{SF_6}$ = 0.72, $Re_c$ = 30 × $10^6$, and $\alpha$ = 1.0 deg.**

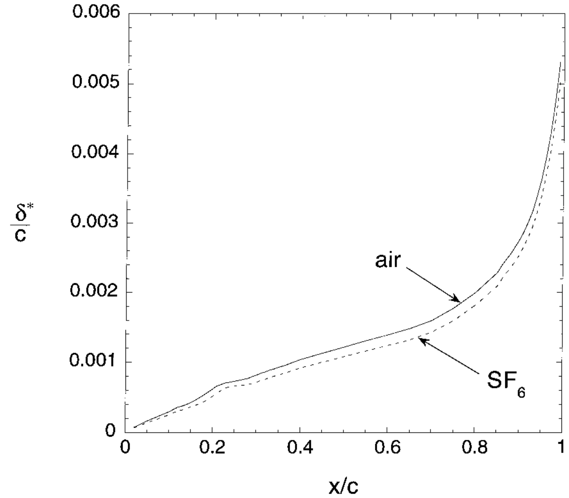



**Fig. 11 Displacement thickness growth, with $M_{N_2}$ = 0.70, $M_{SF_6}$ = 0.72, and $Re_c$ = 30 × $10^6$.**

coefficients are slightly more negative for $SF_6$ than for air (nitrogen) in both the computations and the experiment. The reasons for these differences are not entirely clear, but Anderson[9] noted this same effect in his earlier computational study and indicated that the $SF_6$ boundary layer was appreciably thinner than the air boundary layer, which could result in less upstream movement of the shock as a result of displacement effects. A few comments in the next section on viscous similarity may shed additional light on some of the viscous mechanisms that may be responsible for these differences.

#### Comments on Viscous Similarity

The fact that transonic similarity theory is less successful in the shock–boundary-layer interaction region is not unexpected because it is based on small-disturbance potential flow theory. Clearly, if the boundary layers are different in the two gases, then an inviscid transformation process will not account for this difference.

Calculations of the growth of the displacement thickness on the upper surface of the airfoil for both air and $SF_6$ (again, at the similarity Mach numbers of 0.70 for air and 0.72 for $SF_6$) indicate that the air boundary layer grows at a significantly greater rate (also, see Anderson[9]). Figure 11 shows that for the present airfoil, the displacement thickness at the midchord location is roughly 10% greater for air than for $SF_6$, resulting in a slightly larger effective $\tau$ for the air case. One might assume that this could be accounted for by using an effective profile thickness parameter $\tau$ in Eq. (6) (i.e., both $\tau$ and $\gamma$ are allowed to vary between gases). However, an inspection of Eq. (6) reveals that an increased $\tau_{N_2}$ (say, $\tau_{N_2} = 1.01\,\tau_{SF_6}$) increases the similarity Mach number for $SF_6$. This has the effect of moving the $SF_6$ shock location aft, thus further increasing the disagreement between the two gases in the shock region. For the shock to be moved forward, the $SF_6$ similarity Mach number must be reduced, requiring a reduction in $\tau_{N_2}$ relative to $\tau_{SF_6}$. No justification for this is immediately apparent. More likely, the disagreement in the shock location is due to the response of the thicker air (or nitrogen) boundary layer to the adverse pressure gradient generated by the shock. Again, a potential flow scaling technique will not account for this viscous effect.

The calculation of the displacement thickness growth in Fig. 11 assumes that both the air and $SF_6$ boundary layers are fully turbulent. Unfortunately, transition was not measured in the present experiment, but it is unlikely that there would be enough difference in the transition location between the two gases at the high test Reynolds numbers of the current investigation to significantly offset the results shown in Fig. 11. To understand the viscous flow characteristics of a compressible heavy-gas flow and to develop rules for viscous similarity, detailed boundary-layer investigations have to be conducted that examine the viscous phenomena of transition, separation, and shock–boundary-layer interaction.

## Conclusions

Previous computational studies have shown that transonic similarity theory is successful in transforming transonic airfoil pressure distributions in $SF_6$ to equivalent results in air when the flows are primarily inviscid. However, this earlier work also indicated that the scaling is less successful when viscous effects are included. The current experimental results confirm that airfoil pressure distributions in sulfur hexafluoride scale well with nitrogen results at the similarity Mach number as long as any shocks present are weak. The experimental results also indicate that as shock strength increases, scaling in the shock–boundary-layer interaction region becomes less satisfactory, again in agreement with computation. Finally, the experimental results show that the scaling always yields a shock location for heavy gas that is slightly downstream of the location for nitrogen (air). This mismatch is small, however, and the scaling may be adequate for many applications. Since transonic similarity theory is an inviscid scaling technique, unscaled viscous effects in the shock–boundary-layer interaction region are the likely cause of the residual differences. Overall, these present results indicate that heavy gas may be a practicable technique for increasing the test Reynolds number in existing transonic wind tunnels for flows that are not dominated by strong viscous effects.

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